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In Memoriam: Gennady Alekseevich Leonov

Gennady Alekseevich Leonov
(February 2, 1947–April 23, 2018)

N. V. Kuznetsov
Faculty of Mathematics and Mechanics, St. Petersburg State University, Peterhof, St. Petersburg, Russia
Department of Mathematical Information Technology, University of Jyväskylä, Jyväskylä, Finland

S. Abramovich
State University of New York at Potsdam, USA

A. L. Fradkov
Faculty of Mathematics and Mechanics, St. Petersburg State University, Peterhof, St. Petersburg, Russia
Institute of Problems of Mechanical Engineering RAS, Russia

G. Chen
City University of Hong Kong, Hong Kong SAR, P. R. China

Gennady Alekseevich Leonov passed away on April 23, 2018 after a short battle with grievous illness.

The prolific life of Gennady Leonov as a scholar and educator ended tragically at the zenithal time of his indisputable scientific creativity and academic leadership, bestowing on us the legacy of more than 500 publications, 20 monographs, ten patents, and the diligent supervision of five Doctors of Science (a habilitation degree), 16 Doctors of Philosophy (PhD), and 37 Candidates of Science.

Gennady Leonov earned numerous awards and honors over the course of his career. He was a corresponding member of the Russian Academy of Sciences, a Foreign Member of the

Finnish Academy of Science and Letters, a Council Member of the International Federation of Automatic Control (2011–2017), a Highly Cited Researcher in the Russian Federation — the list of his honors and awards is indeed too long[1] to be fully presented in this brief obituary.

Gennady Leonov was born in St. Petersburg (then Leningrad) on February 2, 1947 into the family of Antonina Andreevna Leonova, an accountant, and Aleksey Michailovich Leonov, a blue-collar worker. Growing up, just like many ambitious youngsters of his generation, Gennady was working daily in a manufactory while studying at night in school and completed his high school education on a fast track (two versus three years). In 1964, he passed the highly competitive entrance exams to become a mathematics major student at the Mathematics and Mechanics Faculty of Leningrad State University, the very place he spent the rest of his life, serving as Dean of the Faculty for 30 years (1988–2018). In 1969, Gennady started his post-graduate studies (*aspirantura* in Russian) under the supervision of Arkadii Kh. Gelig, and defended his thesis in 1971, ahead of schedule. In his dissertation for Candidate of Science, his answer was negative to the famous conjecture of Aizerman [1949] in control theory of the most general form. He joined the newly established Department of Theoretical Cybernetics, headed by Vladimir A. Yakubovich [Abramovich *et al.*, 2015; Fradkov, 2017], as Assistant Professor in 1971, and was quickly promoted to the position of Associate Professor. In 1983, Gennady defended his second dissertation (for Doctor of Science), titled "*Stability in the Whole*", which endowed him the Full Professorship.

By the early 1980s, under the supervision of Gennady Leonov, ten dissertations for Candidate of Science had been defended, where important problems in differential equations and control theory were investigated. One of the directions of Gennady's research work at that time was the development of a mathematical theory for phase-locked loop systems [Gelig *et al.*, 1978], when only heuristic engineering methods existed for investigations. His deep mind and insightful view of mathematics for investigating and solving pivotal real-life problems resulted in a major professional achievement: "*the development of the theory of phase synchronization in radio engineering and communication*", with which a team of researchers, Gennady Leonov included, was awarded the 1986 State Prize of USSR, the second in societal prestige and academic standing that existed in the Soviet Union.

The significance of his contributions to such highly recognized scientific work was undoubtably corroborated during the last decade of the 20th century, when systems of phase synchronization became useful tools of vital importance for ubiquitous integration into various new information systems. In particular, frequency synthesis devices in every modern computer are based on such systems of phase synchronization. Phase-locked loops have also been used in the Global Navigation Satellite System (GLONASS) and the Global Positioning System (GPS). It is fair to say that the effectiveness and precise functioning of such systems would not be possible without utilizing the rigorous mathematical methods developed by Gennady Leonov [Leonov, 2006; Leonov *et al.*, 2012; Leonov & Kuznetsov, 2014; Leonov *et al.*, 2015c, 2015d; Best *et al.*, 2016; Kuznetsov *et al.*, 2017].

In 1988, in the spirit of *perestroika*, Gennady Leonov was elected through an open search process to become Dean of the Mathematics and Mechanics Faculty, a position he held until the very last day of his life. In the 1990s, during the difficult years for Russian science, Gennady was instrumental in preserving the rich traditions of scientific achievements of mathematicians, mechanical engineers, and astronomers of the Faculty. With dedication and determination, he demonstrated an extraordinary ability for preserving the celebrated traditions of the St. Petersburg University in mathematics, while advancing his own research.

[1] Personal webpage: http://www.math.spbu.ru/user/leonov/index_en.html.
Google scholar profile: https://scholar.google.ru/citations?hl=en&user=_zv2pFwAAAAJ.
Mathematics genealogy project: http://www.genealogy.ams.org/id.php?id=105152&fChrono=1.

For one, Gennady proposed to integrate Lyapunov functions into the attractors dimension theory [Leonov, 1991; Leonov & Boichenko, 1992; Boichenko *et al.*, 2005; Kuznetsov, 2016], something that enabled him to obtain exact analytic formulas for the Lyapunov dimension of attractors for a number of well-known dynamical systems (see, e.g. some recent works [Leonov *et al.*, 2015a; Leonov *et al.*, 2016; Leonov, 2017; Kuznetsov *et al.*, 2018]). For another, he solved a problem in the most general form of stabilization of linear differential equations [Leonov, 2001; Leonov & Shumafov, 2010] formulated by Brockett [1999] as an open problem in control theory.

By continuing and expanding the work of Vladimir A. Yakubovich and Viktor A. Pliss, his major academic forefathers, Gennady Leonov was able to establish his own scientific school of control theory, qualitative theory of dynamical systems and their applications in science and engineering. One such application is in the study of chaotic dynamics and hidden attractors. Over the past 40 years, the mathematical theory of chaos grew into one of the most vigorous domains of scholarly research. The significance of this theory is due to the complex phenomena of turbulence in hydrodynamics, the challenging problem of weather prediction, the intrinsic ocean current flows, and the wide appearance of chaotic oscillations in electric circuits and systems. More recently, homoclinic and heteroclinic orbits were found to play an important role in the dynamics of transition to chaos. To this end, Gennady proposed a general methodology (called *Fishing principle*) for investigating such orbits [Leonov, 2012, 2013, 2014; Leonov *et al.*, 2015b]. Consequently, the Fishing principle has provided mathematicians with a universal tool for analytically proving the existence of homoclinic orbits in a number of well-known dynamical systems. Last, but not least, the notion of *hidden attractors* and the development of new mathematical methods for investigating such attractors in classic dynamical systems [Leonov & Kuznetsov, 2013; Kuznetsov & Leonov, 2014; Leonov *et al.*, 2015b; Kuznetsov *et al.*, 2018] have attracted a great deal of interest from researchers in various fields, especially mathematicians, engineers, and physicists [Bragin *et al.*, 2011; Sharma *et al.*, 2015a, 2015b; Dudkowski *et al.*, 2016; Chen *et al.*, 2017; Stankevich *et al.*, 2017]. Largely due to the discovery of hidden attractors in dynamical systems, Gennady Leonov and his former doctoral student Nikolay V. Kuznetsov were proclaimed as the Highly Cited Researchers in the Russian Federation twice over two consecutive years (2016–2017) by Clarivate Analytics.

When compiling the achievements of Gennady Leonov, it is impossible to ignore his administrative talents and contributions. In 2007, in close collaboration with Nikolay V. Kuznetsov, he established a Department of Applied Cybernetics. The goal was to connect fundamental mathematics education at the Faculty of Mathematics and Mechanics with the demands of international collaboration in information technologies that graduates of the Faculty have to fulfill. With this goal, Gennady drew on the experience from the Faculty of Information Technology, University of Jyväskylä, Finland, and the affiliated Technopark established under the auspices of Nokia. Over the last decade, the department has accepted annually about 20 third-year university students; the most successful ones being invited for post-graduate studies and the best of the best were selected for participation in the joint Russia-Finland program of PhD studies, which was organized in cooperation with the Dean of the Faculty of Information Technology, Pekka Neittaanmäki (University of Jyväskylä). Productive collaboration between St. Petersburg State University and University of Jyväskylä has been in place for the last ten years. As a recognition, in 2017, the Finnish Academy of Science and Letters elected Gennady Leonov to be a foreign member.

There is too much to remember about the great mathematician, scientist and administrator Gennady A. Leonov. It is with great respect, immense admiration and profound grief that we celebrate the prolific life of our never-to-be-forgotten friend, colleague and mentor by acknowledging his intellect, talents, kindness, wisdom and acumen. He was a great role model for many, and will continually be so for many more to come.

References

Abramovich, S., Kuznetsov, N. & Leonov, G. [2015] "V.A. Yakubovich — Mathematician, "father of the field", and herald of intellectual democracy in science and society," *IFAC-PapersOnLine* **48**, 1–3.

Aizerman, M. A. [1949] "On a problem concerning the stability in the large of dynamical systems," *Uspekhi Mat. Nauk* **4**, 187–188 (in Russian).

Best, R., Kuznetsov, N., Leonov, G., Yuldashev, M. & Yuldashev, R. [2016] "Tutorial on dynamic analysis of the Costas loop," *Ann. Rev. Contr.* **42**, 27–49.

Boichenko, V., Leonov, G. & Reitmann, V. [2005] *Dimension Theory for Ordinary Differential Equations* (Teubner, Stuttgart).

Bragin, V., Vagaitsev, V., Kuznetsov, N. & Leonov, G. [2011] "Algorithms for finding hidden oscillations in nonlinear systems. The Aizerman and Kalman conjectures and Chua's circuits," *J. Comput. Syst. Sci. Int.* **50**, 511–543.

Brockett, R. [1999] "A stabilization problem," *Open Problems in Mathematical Systems and Control Theory*, eds. Blondel *et al.* (Springer, London), pp. 75–78.

Chen, G., Kuznetsov, N., Leonov, G. & Mokaev, T. [2017] "Hidden attractors on one path: Glukhovsky–Dolzhansky, Lorenz, and Rabinovich systems," *Int. J. Bifurcation and Chaos* **27**, 1750115-1–9.

Dudkowski, D., Jafari, S., Kapitaniak, T., Kuznetsov, N., Leonov, G. & Prasad, A. [2016] "Hidden attractors in dynamical systems," *Phys. Rep.* **637**, 1–50.

Fradkov, A. [2017] "Scientific school of Vladimir Yakubovich in the 20th century," *IFAC-PapersOnLine* **50**, 5231–5237.

Gelig, A., Leonov, G. & Yakubovich, V. [1978] *Stability of Nonlinear Systems with Nonunique Equilibrium* (in Russian) (Nauka); English transl. [2004] *Stability of Stationary Sets in Control Systems with Discontinuous Nonlinearities* (World Scientific, Singapore).

Kuznetsov, N. & Leonov, G. [2014] "Hidden attractors in dynamical systems: Systems with no equilibria, multistability and coexisting attractors," *IFAC Proc. Vol.* **47**, 5445–5454.

Kuznetsov, N. [2016] "The Lyapunov dimension and its estimation via the Leonov method," *Phys. Lett. A* **380**, 2142–2149.

Kuznetsov, N., Leonov, G., Yuldashev, M. & Yuldashev, R. [2017] "Hidden attractors in dynamical models of phase-locked loop circuits: Limitations of simulation in MATLAB and SPICE," *Commun. Nonlin. Sci. Numer. Simul.* **51**, 39–49.

Kuznetsov, N., Leonov, G., Mokaev, T., Prasad, A. & Shrimali, M. [2018] "Finite-time Lyapunov dimension and hidden attractor of the Rabinovich system," *Nonlin. Dyn.* **92**, 267–285.

Leonov, G. [1991] "On estimations of Hausdorff dimension of attractors," *Vestnik St. Petersburg University*: *Mathematics* **24**, 38–41; Transl. from Russian [1991] *Vestnik Leningradskogo Universiteta, Mathematika* **24**, 41–44.

Leonov, G. & Boichenko, V. [1992] "Lyapunov's direct method in the estimation of the Hausdorff dimension of attractors," *Acta Appl. Math.* **26**, 1–60.

Leonov, G. A. [2001] "On the Brockett stabilization problem," *Dokl. Phys.* **46**, 268–270.

Leonov, G. [2006] "Phase-locked loops. Theory and application," *Automat. Rem. Contr.* **10**, 1573–1609.

Leonov, G. & Shumafov, M. [2010] "Stabilization of controllable linear systems," *Nonlin. Dyn. Syst. Th.* **10**, 235–268.

Leonov, G. [2012] "General existence conditions of homoclinic trajectories in dissipative systems. Lorenz, Shimizu–Morioka, Lu and Chen systems," *Phys. Lett. A* **376**, 3045–3050.

Leonov, G., Kuznetsov, N., Yuldashev, M. & Yuldashev, R. [2012] "Analytical method for computation of phase-detector characteristic," *IEEE Trans. Circuits Syst.-II*: *Express Briefs* **59**, 633–647.

Leonov, G. [2013] "The Tricomi problem on the existence of homoclinic orbits in dissipative systems," *J. Appl. Math. Mech.* **77**, 296–304.

Leonov, G. & Kuznetsov, N. [2013] "Hidden attractors in dynamical systems. From hidden oscillations in Hilbert–Kolmogorov, Aizerman, and Kalman problems to hidden chaotic attractors in Chua circuits," *Int. J. Bifurcation and Chaos* **23**, 1330002-1–69.

Leonov, G. [2014] "Fishing principle for homoclinic and heteroclinic trajectories," *Nonlin. Dyn.* **78**, 2751–2758.

Leonov, G. & Kuznetsov, N. [2014] *Nonlinear Mathematical Models of Phase-Locked Loops. Stability and Oscillations* (Cambridge Scientific Publisher).

Leonov, G., Alexeeva, T. & Kuznetsov, N. [2015a] "Analytic exact upper bound for the Lyapunov dimension of the Shimizu–Morioka system," *Entropy* **17**, 5101–5116.

Leonov, G., Kuznetsov, N. & Mokaev, T. [2015b] "Homoclinic orbits, and self-excited and hidden attractors in a Lorenz-like system describing convective fluid motion," *Eur. Phys. J. Special Topics* **224**, 1421–1458.

Leonov, G., Kuznetsov, N., Yuldashev, M. & Yuldashev, R. [2015c] "Hold-in, pull-in, and lock-in ranges of PLL circuits: Rigorous mathematical definitions and limitations of classical theory," *IEEE Trans. Circuits Syst.-I*: *Regular Papers* **62**, 2454–2464.

Leonov, G., Kuznetsov, N., Yuldashev, M. & Yuldashev, R. [2015d] "Nonlinear dynamical model of Costas loop and an approach to the analysis of its stability in the large," *Sign. Process.* **108**, 124–135.

Leonov, G., Kuznetsov, N., Korzhemanova, N. & Kusakin, D. [2016] "Lyapunov dimension formula for the global attractor of the Lorenz system," *Commun. Nonlin. Sci. Numer. Simul.* **41**, 84–103.

Leonov, G. [2017] "Generalized Lorenz equations for acoustic-gravity waves in the atmosphere. Attractors dimension, convergence and homoclinic trajectories," *Commun. Pure Appl. Anal.* **16**, 2253–2267.

Sharma, P., Shrimali, M., Prasad, A., Kuznetsov, N. & Leonov, G. [2015a] "Control of multistability in hidden attractors," *Eur. Phys. J. Special Topics* **224**, 1485–1491.

Sharma, P., Shrimali, M., Prasad, A., Kuznetsov, N. & Leonov, G. [2015b] "Controlling dynamics of hidden attractors," *Int. J. Bifurcation and Chaos* **25**, 1550061-1–7.

Stankevich, N., Kuznetsov, N., Leonov, G. & Chua, L. [2017] "Scenario of the birth of hidden attractors in the Chua circuit," *Int. J. Bifurcation and Chaos* **27**, 1730038-1–18.

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OBITUARY: GENNADY ALEKSEEVICH LEONOV (1947-2018)

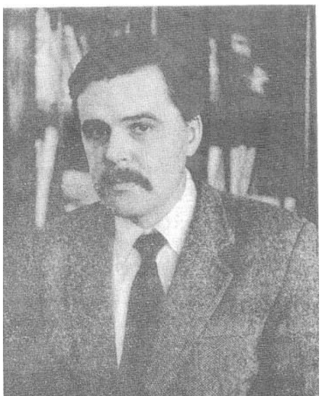
Sergei Abramovich, *State Universtity of New York at Potsdam, United States*
abramovs@potsdam.edu
Nikolay V. Kuznetsov, *St. Petersburg State University, Russia and University of Jyväskylä, Finland*
nkuznetsov239@gmail.com
Pekka Neittaanmäki, *University of Jyväskylä, Finland*
pekka.neittaanmaki@jyu.fi

Abstract

The paper is written in memoriam of G. A. Leonov – the founding member of the *Open Mathematical Education Notes* Editorial Advisory Board, Dean (1988-2018) of the Faculty of Mathematics and Mechanics of St. Petersburg State University, the author/co-author of 20 books and more than 500 articles. The most important events of his life are considered through the lenses of mathematics education.

Key words: differential equations, control theory, mathematics education.

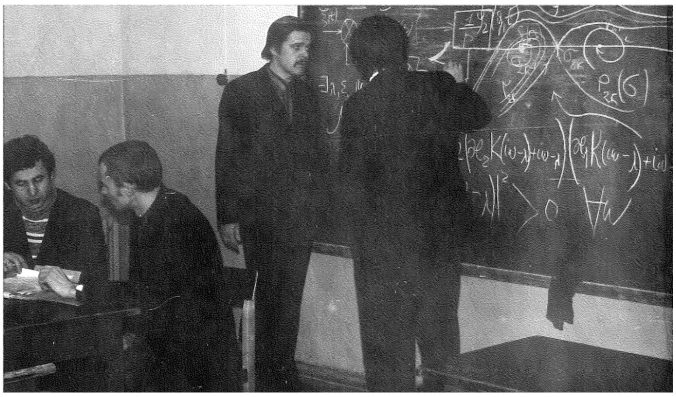
ZDM Subject Classification: **A30, A40, U90**



Gennady Alekseevich Leonov, a founding member of the *Open Mathematical Education Notes* Editorial Advisory Board, passed away on April 23, 2018 after a short battle with a grievous illness. His prolific life of Scholar and Teacher, which tragically ended at the zenith of his indisputable scientific creativity and unparalleled academic leadership, bestowed upon us the remarkable legacy of more 500 publications, 20 monographs, 10 patents along with the record of diligent supervision of five Doctors of Science (a habilitation degree), 16 Doctors of Philosophy

(PhD), and 37 Candidates of Science. Corresponding member of Russian Academy of Sciences, a bearer of many notable awards and well-deserved recognitions, a Council Member of International Federation of Automatic Control (2011-2017), a Foreign Member of the Finnish Academy of Science and Letters, the Highly Cited Mathematician of Russian Federation – the list of Professor Leonov's honors and decorations is too long to be fully presented in this brief obituary.

Gennady Leonov was born in St. Petersburg (then Leningrad) on February 2, 1947 in the family of Antonina Andreevna Leonova, an accountant, and Aleksey Michailovich Leonov, a blue-collar worker. Just as many ambitious youths of his generation, Gennady combined factory work during the day with studies in the evening to allow for a fast track (two vs. three years) graduation from high school followed by highly competitive enrollment (1964) into the cohort of mathematics students at the Mathematics and Mechanics Faculty of Leningrad State University, the very place he would later be Dean for 30 years (1988-2018). In 1969, Gennady began his post-graduate studies (aspirantura in Russian) within the scientific group of V. A. Yakubovich [1] at the V. I. Smirnov Research Institute of Mathematics and Mechanics of Leningrad State University. In 1971 he defended, ahead-of-schedule, his Candidate of Science dissertation in which a negative answer to a famous in control theory conjecture by Aizerman [3] was given in the most general case. Gennady then joined a newly developed Department of Theoretical Cybernetics (chaired by V. A. Yakubovich) as assistant professor in 1971 and was soon promoted to associate professor. Already by the early 1980s, under the direction of Professor Leonov, ten Candidate of Science dissertations have been defended in which important problems of differential equations and control theory were investigated. In 1983, Gennady Leonov defended the second (Doctor of Science) dissertation titled "Stability in the Whole", something that enabled his appointment as full professor.



Leningrad, 1976. After a seminar, working on a joint paper [2].
Near the board: G. Leonov (left) and Yu. Koryakin. At the desk: S. Abramovich (left) and V. Reitmann.

Gennady Leonov authored/co-authored many books on differential equations and control theory. The first major book *Stability of Stationary Sets in Control Systems with Discontinuous Nonlinearities* (English edition in 2004 by World Scientific [19]), was written jointly with A. Kh. Gelig and V. A. Yakubovich. Leonov's chapter in this book was devoted to the development of mathematical theory of phase-locked loop systems for which only non-rigorous engineering methods of investigation existed. His insightful awareness of mathematical methods as tools of investigation of pivotal real-life problems resulted in the major scientific achievement: as stated in the award citation, for "the development of the theory of phase synchronization in radio engineering and communication" a team of researchers, Gennady Leonov included, was awarded a 1986 State Prize of USSR, the second by the societal prestige and the academic standing award that existed in the Soviet Union.

The significance of Leonov's contribution to such highly recognized research work was indubitably corroborated during the last decade of the 20th century when the systems of phase synchronization suddenly gained the status of tools of vital importance due to their ubiquitous integration into new information systems. In particular, frequency synthesis devices present in every modern computer are based on such systems of phase synchronization. Phase-locked loops have also been used in the Global Navigation Satellite System (GLONASS) and the Global Positioning System (GPS). The effectiveness and error-free functioning of such systems would not be possible without designers' utilization of rigorous mathematical methods developed by Gennady Leonov.

On the merits of his outstanding research, Professor Leonov had secured a rightful place in the St. Petersburg school of control theory, the eminent leaders of which have been Corresponding Members of Russian Academy of Sciences A. I. Lur'e (1901-1980), V. A. Yakubovich (1926-2012), and V. I. Zubov (1930-2000). He was a member of St. Petersburg regional group of the Russian National Committee of Automatic Control chaired by Academician V. G. Peshehonov. In 2011, upon the recommendation of Academician A. B. Kurzhanski, Chairman of Russian National Committee of Automatic Control, he was elected to the Council of International Federation of Automatic Control and served full two-term limits (2011-2017) there.

Leningrad, 1996. At Yakubovich's residence.
From left to right: G. Leonov, V. Yakubovich, A. Pervozvansky, and A. Fradkov.

Japan, 2012. IFAC Council meeting.
G. Leonov (left), A. Kurzhanski (center), and N. Kuznetsov.

By carrying on and expanding work by V. A. Yakubovich and V. A. Pliss, his major academic forefathers, and, at the same time, drawing on the ideas rooted in the school of A. A. Andronov [4], Professor Leonov was able to establish his own scientific school of control theory, qualitative theory of dynamical systems and their applications in science and engineering. He was instrumental in bringing to fruition his insightful proposal of splitting the study of the problems of cybernetics in two parallel tracks – theoretical cybernetics, dealing mostly with the synthesis and adaptation of new systems, and applied cybernetics, concerned with the rigorous study of the existing systems. As a result, in 2007, in close collaboration with N. V. Kuznetsov, he established a new Department of Applied Cybernetics of which the former became the first academic appointee.

The importance of the applied cybernetics track can be illustrated through the paper by Leonov and Kuznetsov [13] (memorable in a sense that it was submitted for publication two hours before the sudden death of the first author) on the study of flutter suppression in the design of aircraft. The genesis of research on the gravity of the avoidance of flutter – an oscillation often destructive for an aircraft, when the latter absorbs energy from the airstream – can be found in a classic research by Mstislav Keldysh (1911-1978), President of the Soviet Union Academy of Sciences during 1961-1975, who, while being successful in the design of aircraft free from flutter, nonetheless admitted that his applied mathematics research work does not offer a rigorous mathematical proof but, rather, it provides aircraft designers with a number of conclusions based on intuitive considerations. Just as in the case of the systems of phase synchronization, it is by using research tools developed within the Department of Applied Cybernetics chaired by Professor Leonov that a combination of rigorous analytical and reliable numerical methods allowed for a mathematically thorough confirmation of Keldysh's intuitive considerations enabling the avoidance of flutter in aeronautics.

The above-mentioned case of rigorous flutter research (made possible due to the modern methods of investigation, unfortunately not available in Keldysh's era) is just one of many instances demonstrating that Professor Leonov possessed an acute sense of the importance of historical roots in the contemporary study of applied control systems. In his books [5-8, 10, 12, 15-17, 19], he built the state-of-the-art discussion of the behavior of trajectories of dynamical systems on the classic studies of the theory of differential equations by Aleksandr Lyapunov (1857-1918, Russia), James Maxwell (1831-1879, Scotland), Oskar Perron (1880-1975, Germany), Henri Poincare (1854-1912, France), Francesco Tricomi (1897-1978, Italy), Balthasar van der Pol (1889-1959, Holland), Vito Volterra (1860-1940, Italy), Ivan Vyshnegradsky (1832-1895, Russia), Nikolay Zukovsky (1847-1921, Russia) and other scholars. From an educational point of view, such thoughtful attention to historical perspectives in the development of new ideas is extremely important for it demonstrates to students that the modern-day scientific advances would not be possible without taking into consideration the past experience as the psychological foundation of productive thinking.

In 1988, in the spirit of perestroika, Professor Leonov was elected through an open search process to become Dean of Mathematics and Mechanics Faculty, a position he held continuously till the very last day of his life. In the 1990s, during difficult years for Russian science, Professor Leonov was instrumental in preserving rich traditions of scientific achievements of mathematicians, mechanical engineers, and astronomers of the Faculty. At the same time, Professor Leonov was successful being not only the major custodian of celebrated traditions of the St Petersburg University mathematics, but also persevering his own research advancement. As Dean, he paid great attention to the issues of mathematics education at all levels. At the tertiary level, the main pedagogical goal of the formation of the Department of Applied Cybernetics was to connect fundamental mathematics education at the Faculty of Mathematics and Mechanics with the demands of international companies of information technologies (IT) that graduates of the Faculty working in that area have to satisfy. During the era of educational innovations, the preparation of qualified computer programmers is impossible without offering field experience in software engineering. Furthermore, students of mathematics with apprenticeship-like experience working for an IT company have come to possess a high-level appreciation of theoretical courses that deal with the issues of governance of IT projects and the quality of the development of programming products [11].

Such revision of the traditional preparation of mathematicians that connects theory and practice can be described in terms of the modern-day signature pedagogy construct – the "types of teaching that organize the fundamental ways in which future practitioners are educated for their professions" [18, p. 52]. To this end, Professor Leonov drew on the experience of Professor Neittaanmäki of the University of Jyväskylä (Finland) Faculty of Information Technology.

Productive collaboration between St. Petersburg State University and University of Jyväskylä towards the joint preparation of highly qualified workforce for the frontiers of information technology research has been in place for the last ten years and resulted in more than ten successful PhD defenses. As a result, in 2017, the Finnish Academy of Science and Letters elected Professor Leonov as its foreign member, one of only six representatives from the Russian Federation.

Jyväskylä, Finland, 2011. After a PhD defense.
From left to right: N. Kuznetsov, G. Leonov, S. Abramovich, O. Kuznetsova, P. Neittaanmäki.

Jyväskylä, Finland, 2016. A multinational collaboration.
G. Leonov with P. Neittaanmäki (left) and S. Abramovich.

Helsinki, 2017. Professor Leonov, Member of Finnish Academy of Science and Letters.

Professor Leonov had strong belief that advances in the development of mathematical sciences are the result of studying concrete problems. In [6], he advised the learners of control theory that it is the study of concrete control systems that motivated the development of mathematical machinery needed for solving such problems. At the same time, the power of tools developed for solving concrete problems at the level of rigor enabled far-reaching generalizations of those tools to make them applicable to solving a multitude of problems both within and outside mathematics. In [7], he argued that a student is interested in studies and appreciates learning as an intellectual endeavor if he or she is confident in the usefulness of material to be studied. The usefulness implies applicability. That is, Leonov's pedagogical motto was the importance of concrete problems as pedagogical tools in the teaching of mathematics. Furthermore, he demonstrated [7] that experience in the study of concrete systems makes it possible to carry out qualitative analysis of many complex systems without presentation of their formal mathematical models. For example, a dynamical system in which its every subsystem develops with a positive derivative, the effect of instability is subdued. That is, a continuous growth within a system implies stability of the system's functioning. This principle can inform the policy of management – try not demote employees but only promote them. That is, any element of a system that moves only forward is a guarantee of the system's desirable stable behavior. Such a qualitative understanding of the stability of functioning of a dynamical system has important implication for the success of the enterprise of mathematics education: never blame a student for an erroneous answer but instead, turn an incorrect answer into a thinking device keeping in mind that one of the main responsibilities of a mathematics teacher at any educational level is to encourage rather than to suppress a mathematical discourse.

In order for such a discourse to flourish at the pre-college level, Professor Leonov always strived to provide conditions for continuous uplifting of mathematically talented high schoolers, both from urban and rural areas of Russian Federation [9]. Those conditions included the availability of summer camps and, most importantly, a reliable cohort of sponsors financially supporting diverse students' participation in programs offered by the camps. Also, he paid special attention to the work of boarding schools and other already established centers of preparation of youths with interest in STEM disciplines [14].

To conclude, it is with great respect, immense admiration, and profound grief that we celebrate the prolific life of our never-to-be-forgotten Friend, Teacher, and Colleague by acknowledging his intellect, talents, kindness, wisdom and acumen, while expressing confidence that his qualities will serve as a guiding star for anyone with true aspirations to become a productive member of the modern society.

References

[1] Abramovich, S., Kuznetsov, N. V., and Leonov, G. A. (2015). V. A. Yakubovich – mathematician, "father of the field", and herald of intellectual democracy in science and society. *IFAC-PapersOnLine*, **48**(11): 1-3. Available at http://www.sciencedirect.com/science/article/pii/S2405896315012318.

[2] Abramovich, S., Leonov, G., Koriakin, Yu., and Reitmann, V. (1977). Frequenzbedingungen fur Schwingungen in Discreten Systemen. II. *Wiss. Z. Techn. Univer. Dresden*, **26** (1): 115-122.

[3] Aizerman, M. A. (1949). On a problem concerning the stability "in the large" of dynamical systems. *Russian Mathematical Surveys*, **4**(4):186–188. (In Russian).

[4] Andronov, A. A., Vitt, A. A., and Khaikin, S. E. (1966). *Theory of Oscillators* (translated from Russian by F. Immirzi). Oxford: Pergamon Press.

[5] Boichenko, V. A., Leonov, G. A., and Reitmann, V. (2005). *Dimension Theory for Ordinary Differential Equations*. Springer: Teubner.

[6] Leonov, G. A. (2001). *Mathematical Problems of Control Theory: An Introduction*. Singapore: World Scientific.

[7] Leonov, G. A. (2006). *Control Theory*. St. Petersburg, Russia: St. Petersburg University Press. (In Russian).

[8] Leonov, G. A. (2008). *Strange Attractors and Classical Stability Th*eory. St. Petersburg, Russia: St. Petersburg University Press.

[9] Leonov, G. A. (2012). About mathematics education in Russia and St. Petersburg: The past, the present and the future. *Differential Equations and Control Processes*, **2**: 4-8. (In Russian).

[10] Leonov, G. A., Burkin, I. M., and Shepeljavyi, A. I. (1996). *Frequency Methods in Oscillation Theory*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

[11] Leonov, G. A., Kiyaev, V. I., Kuznetsov, N. V., Onossovski, V. V., and Seledzhi, S. M. (2012). Computers and software engineering: Developing new models for educating mathematicians. In S. Abramovich (Ed.), *Computers in Education*, vol. 2 (pp. 157-169). Hauppauge, NY: Nova Science Publishers.

[12] Leonov, G. A., and Kuznetsov, N. V. (2014). *Nonlinear Mathematical Models of Phase-Locked Loops. Stability and Oscillations.* Cambridge, UK: Cambridge Scientific Press.

[13] Leonov, G. A., and Kuznetsov, N. V. (2018, accepted). On the suppression of flatter in the Keldysh model. *Doklady Physics*.

[14] Leonov, G. A., Kuznetsov, N. V., Kudryashova, E. V., and Aleksandrov, K. D. (2018). *Introduction into Mathematics. Part 1: Axioms and Theorems*. St Petersburg: Lema. (In Russian).

[15] Leonov G. A., Ponomarenko D. V., and Smirnova V. B. (1996). *Frequency-Domain Methods for Nonlinear Analysis. Theory and Applications*. Singapore: World Scientific.

[16] Leonov, G. A., Reitmann, V., and Smirnova, V. B. (1992). *Non-Local Methods for Pendulum-Like Feedback Systems*. Teubner-Texte zur Mathematik, Band 132. Stuttgart-Leipzig: Teubner.

[17] Leonov, G. A., and Shumafov, M. M. (2012). *Stabilization of Linear Systems*. Cambridge, UK: Cambridge Scientific Publishers.

[18] Shulman, L. S. (2005). Signature pedagogies in the professions. *Daedalus*, **134**(3): 52-59.

[19] Yakubovich, V. A., Leonov, G. A., and Gelig, A. Kh. (2004). *Stability of Stationary Sets in Control Systems with Discontinuous Nonlinearities*. Singapore: World Scientific.

ХРОНИКА

Памяти Геннадия Алексеевича Леонова

23 апреля 2018 г. на 72-м году жизни после тяжелой болезни скончался декан математико-механического факультета СПбГУ, заведующий кафедрой прикладной кибернетики СПбГУ, заведующий лабораторией информационно-управляющих систем Института проблем машиноведения РАН, член-корреспондент РАН, иностранный член Финской академии наук и литературы, доктор физико-математических наук, профессор Геннадий Алексеевич Леонов.

Геннадий Алексеевич Леонов закончил математико-механический факультет Ленинградского государственного университета в 1969 году и был принят в аспирантуру.

В своих первых научных работах Г. А. Леонов развивал подходы В. А. Якубовича, В. А. Плисса и Горьковской научной школы А. А. Андронова. В 1971 г. он защитил кандидатскую диссертацию на тему «Глобальная устойчивость систем управления», а в 1983 году — докторскую диссертацию «Устойчивость в целом». С 1971 года Г. А. Леонов работал в ЛГУ ассистентом, доцентом, профессором. В 1986 году он получил звание профессора и был назначен на должность проректора ЛГУ. С 1988 года и до последних дней жизни Геннадий Алексеевич Леонов был деканом математико-механического факультета.

Г. А. Леонов — специалист в области теории управления, теории устойчивости, нелинейных колебаний и теории синхронизации электромеханических и электронных систем, автор более 470 научных работ, в том числе 22 монографий. Им создана всемирно известная научная школа, где разработаны новые математические методы и решены трудные математические задачи, важные для создания новых технологий в системах управления, электронных и информационных системах, аэрокосмической технике.

Под его руководством были разработаны новые методы анализа и синтеза дифференциальных, интегральных, интегро-дифференциальных уравнений и дискретных динамических систем с цилиндрическим фазовым пространством, которые описывают математические модели систем фазовой синхронизации, используемых в телекоммуникациях, компьютерных архитектурах и системах глобальной навигации.

В 1991 году Г. А. Леонов предложил использовать функции Ляпунова для оценки размерности аттракторов. Это позволило доказать гипотезу Альпа Идена о максимуме локальной ляпуновской размерности и впервые получить аналитически точные формулы ляпуновской размерности аттракторов для ряда известных динамических систем.

В 2000 году Г. А. Леоновым решена проблема нестационарной стабилизации, которую поставил известный ученый Роджер Брокетт (Гарвардский университет, США) в книге «Open Problems in Mathematical Systems and Control Theory» (1999 г.).

В 1987 году Г. А. Леоновым были впервые получены необходимые и достаточные условия существования гомоклинической траектории в знаменитой системе Лоренца, а в 2012 году сформулирован общий принцип: «принцип рыбака» (fishing principle) для исследования гомоклинических и гетероклинических траекторий, которые играют важную роль в сценариях перехода к хаосу. Этот принцип позволил впервые провести универсальное рассуждение, давшее возможность получить аналитическое доказательство существования таких траекторий для ряда известных динамических систем.

В 2010 году Г. А. Леоновым и его учеником Н. В. Кузнецовым было введено новое понятие в теории колебаний: «скрытые колебания» (hidden oscillations). Ими были разработаны новые математические методы исследования таких колебаний, позволившие обнаружить скрытые колебания в различных системах автоматического регулирования, механических и физических моделях. Эти исследования привлекли внимание научного сообщества, и первые публикации научной школы Г. А. Леонова по этой тематике в 2016 году вошли в 1% самых высокоцитируемых статей библиометрической базы Web of Science и стали самыми цитируемыми статьями в известных журналах: Journal of Computer and Systems Sciences International (переводной версии журнала «Известия РАН. Теория и системы управления»); Physics Letters A; Physica D: Nonlinear Phenomena; International Journal of Bifurcation and Chaos in Applied Sciences and Engineering. В 2016 году обзорная статья по тематике скрытых колебаний была опубликована в одном из самых престижных высокорейтинговых журналов — Physics Reports. Во многом благодаря интересу к этой тематике Г. А. Леонов и Н. В. Кузнецов были названы компанией Clarivate Analytics самыми высокоцитируемыми российскими учеными (Russian Highly Cited Researchers) в области математики в 2016 и 2017 годах.

Теоретические исследования Г. А. Леонова успешно использовались для решения важных прикладных задач. После крупнейшей техногенной катастрофы на Саяно-Шушенской ГЭС Г. А. Леонов с учениками исследовал причины аварии. В 2015 году была построена математическая динамическая модель, учитывающая совместную работу генератора, турбины и системы управления агрегата Саяно-Шушенской ГЭС, в которой было показано возникновение нежелательных опасных колебаний.

Последняя научная работа Г. А. Леонова была посвящена строгому анализу подавления флаттера и стабилизации в модели академика М. В. Келдыша*. В этой

*Слайды пленарного доклада «On the Suppression of Flutter in the Keldysh Model» (G. Leonov, N. Kuznetsov) на конференции «Поляховские чтения» (Санкт-Петербург, 2018) http://www.math.spbu.ru/user/nk/PDF/2018-PR-plenary-Flutter-suppression-Keldysh-model.pdf работе для решения задачи стабилизации и получения критерия глобальной устойчивости были развиты методы частотного анализа систем управления с разрывными характеристиками.

Г. А. Леонов является ярким представителем Петербургской школы теории автоматического управления, становление которой связано с именами таких выдающихся ученых, как А. И. Лурье (1901–1980), В. А. Якубович (1926–2012), В. И. Зубов (1930–2000). В течение многих лет Г. А. Леонов был членом бюро Национального комитета по автоматическому управлению, и в 2011–2017 гг. представлял Российскую Федерацию в Совете Международной федерации по автоматическому управлению (IFAC).

Г. А. Леонов был членом Национального комитета по теоретической и прикладной механике, членом правления Петербургского математического общества. С 2016 года он возглавлял Федеральное учебно-методическое объединение в сфере высшего образования по компьютерным и информационным наукам.

Г. А. Леонов — лауреат университетской премии ЛГУ (1985 г.) за цикл научных работ по устойчивости нелинейных динамических систем, Государственной премии СССР (1986 г.) за математическую теорию фазовой синхронизации, премии Технического университета Дрездена (1989 г.) за работы по хаотической динамике, премии им. А. А. Андронова РАН (2012 г.) за цикл работ «Развитие методов синхронизации и анализа периодических и хаотических колебаний в коллективных системах автоматического фазового управления», премии им. П. Л. Чебышёва Правительства Санкт-Петербурга и Санкт-Петербургского научного центра РАН (2015 г.) за фундаментальный вклад в разработку понятий и методов анализа динамических систем. Заслуженный работник Высшей школы РФ (1999 г.), в 2005 г. получил благодарность Президента РФ за большой вклад в подготовку высококвалифицированных специалистов и многолетнюю плодотворную деятельность, награжден Орденом Дружбы (2007 г.), в 2011 г. получил Медаль Университета Ювяскюля (University of Jyväskylä, Finland) за выдающиеся достижения в области математики, награжден Орденом Почета (2014 г.).

Г. А. Леонов всегда много и плодотворно работал, успешно сочетая научную работу с административной [1–9]. Как в своей научной работе Г. А. Леонов особое внимание уделял известным классическим задачам и их решению с помощью современных подходов, так и в административной работе он сочетал сохранение традиций с острым чувством необходимости перемен и развития. Возглавляемый им в течение 30 лет, включая трудные 1990-е годы, математико-механический факультет сохранил и продолжил славные традиции математической, механической и астрономической научных школ Петербургского университета. При нем на факультете было организовано отделение информатики, где созданы новые кафедры: системного программирования, теории параллельных алгоритмов, информационно-аналитических систем. Г. А. Леонов уделял большое внимание глубокой математической подготовке программистов-выпускников факультета, что позволило им занять лидирующие позиции во многих ИТ-компаниях и исследовательских центрах. В 2000 году студенты математико-механического факультета СПбГУ первыми из российских студентов стали чемпионами мира по программированию, а потом повторили этот результат в 2001, 2014 и 2016 годах, много раз входили в число призеров этих престижных соревнований. В декабре 2006 года Г. А. Леонов создал и возглавил новую кафедру прикладной кибернетики. Именно ее выпускники в 2013 году, по поручению ректора СПбГУ, первыми подготовили и защитили диссертации на степень Ph.D. СПбГУ под руководством Геннадия Алексеевича Леонова.

Сотрудники математико-механического факультета СПбГУ навсегда сохранят память о выдающемся ученом, доброжелательном и требовательном учителе, соратнике и друге Геннадии Алексеевиче Леонове.

1. Список научных работ Г. А. Леонова: на портале eLIBRARY https://elibrary.ru/author_items.asp?authorid=4556, на портале Google Scholar https://scholar.google.ru/citations?user=_zv2pFwAAAAJ
2. Научная школа Г. А. Леонова. Фильм цикла «Матрица науки» (подготовлен по заказу Комитета по науке и высшей школе Санкт-Петербурга в 2016 году): https://www.youtube.com/watch?v=X3bla8IYcvk
3. Заявление научных руководителей по случаю первой защиты Ph.D. SPbSU, Кузнецов Н. В., Леонов Г. А., Neittaanmäki P., 10 июля 2013, http://www.math.spbu.ru/user/nk/PDF/2013-First-PhD-SPbSU-Renat-Yuldashev-Supervisors.pdf
4. Г. А. Леонов. Ускоренное научно-техническое развитие в условиях санкций, Вестник Российской Академии Наук, 86(3), 2016, стр. 280.
5. Г. А. Леонов. О математическом образовании в России и Санкт-Петербурге. Прошлое, настоящее, будущее, Дифференциальные уравнения и процессы управления, №2, 2012, стр. 4–8 http://www.math.spbu.ru/diffjournal/pdf/2012_math_education_leonov.pdf
6. Выступление Г. А. Леонова с докладом «В. А. Якубович и Санкт-Петербургская математическая школа» на совместном заседании Санкт-Петербургского математического общества и Санкт-Петербургского семинара по теории управления 23 июня 2015 года: https://youtu.be/bXzXAxutiyM (по материалам статьи: S. Abramovich, N. Kuznetsov, G. Leonov, V. A. Yakubovich — mathematician, "father of the field", and herald of intellectual democracy in science and society, IFAC-PapersOnLine, 48(11), 2015, 1–3 https://www.sciencedirect.com/science/article/pii/S2405896315012318).
7. Г. А. Леонов, А. Н. Терехов, Б. А. Новиков, Е. А. Крук, В. М. Нестеров. Создание на математико-механическом факультете СПбГУ научно-образовательного ИТ-кластера на базе современной фундаментальной математики, Компьютерные инструменты в образовании, №2, 2017, 42–57.
8. Г. А. Леонов, М. Х. Немешев. О реализации стратегии Президента РФ в подготовке IT-специалистов мирового уровня, Дифференциальные уравнения и процессы управления, №1, 2018, стр. 126–141. http://www.math.spbu.ru/diffjournal/pdf/2012_math_education_leonov.pdf
9. Г. А. Леонов, Н. В. Кузнецов, Е. В. Кудряшова, К. Д. Александров. Введение в математику для школьников и студентов. Часть 1. Аксиомы и теоремы. СПб.: Изд-во Лема, 2018. 128 с.

А. А. Архипова, А. К. Беляев, С. М. Бауэр, Е. Б. Воронкова, С. В. Востоков, В. А. Гаген-Торн, А. Х. Гелиг, Ю. К. Демьянович, С. М. Ермаков, О. А. Иванов, Д. А. Индейцев, Н. В. Кузнецов, Е. В. Кустова, С. К. Матвеев, Е. Г. Михайлова, В. А. Морозов, Н. Ф. Морозов, А. И. Назаров, В. Б. Невзоров, В. М. Нежинский, Я. Ю. Никитин, Б. А. Новиков, В. А. Плисс, С. Ю. Пилюгин, А. И. Разов, И. В. Романовский, В. М. Рябов, А. С. Савельев, А. Л. Смирнов, А. Н. Терехов, П. Е. Товстик, Н. Н. Уральцева, А. Л. Фрадков, К. В. Холшевников, Н. А. Широков

Персональная страница Г.А. Леонова на сайте математико-механического факультета СПбГУ

Список публикаций Г.А. Леонова (уточненная библиография)

Книги

1. Leonov G.A., Shumafov M.M., Stabilization of linear system, Cambridge Scientific Publishers, 2012.
2. Леонов Г.А., Шумафов М.М., Тешев В.А., Устойчивость систем с гистерезисом, Изд-во АГУ, 2012.
3. Леонов Г.А., Кондратьева Н.В., Анализ устойчивости электрических машин переменного тока, Изд-во Санкт-Петербургского университета, 2009.
4. Leonov G.A., Strange attractors and classical stability theory, St. Petersburg State University Press, 2008.
5. Леонов Г.А., Теория управления. Изд-во Санкт-Петербургского университета, 2006. Сер. "Петербургская классика".
6. Леонов Г.А. Хаотическая динамика и классическая теория устойчивости движения. Москва - Ижевск, 2006. Сер. "Современная математика".
7. Леонов Г.А., Шумафов М.М. Методы стабилизации линейных управляемых систем. Изд-во Санкт-Петербургского университета, 2005.
8. Boichenko V.A., Leonov G.A., Reitmann V., Dimention Theory for Ordinary Differential Equation. Ser. Teubner-Texte zur Mathematik, Springer, 2005.
9. Yakubovich V.A., Leonov G.A., Gelig A.Kh., Stability of Stationary Sets in Control Systems with Discontinuous Nonlinearities. Singapore, World Scientific, 2004.
10. Леонов Г.А, Селеджи С.М. Системы фазовой синхронизации в аналоговой и цифровой схемотехнике. Спб, "Невский диалект", 2002.
11. Leonov G.A., Mathematical Problems of Control Theory. An Introduction. Singapore, World Scientific, 2001. Series on Stability, Vibration and Control of Systems, Series A: Volume 6.
12. Леонов Г.А., Смирнова В.Б., Математические проблемы теории фазовой синхронизации. М., "Наука", 2000.
13. Леонов Г.А., Математические проблемы теории управления. Мотивация к анализу. Изд-во Санкт-Петербургского университета, 1999.
14. Leonov G.A., Ponomarenko D.V, Smirnova V.B., Frequency-Domain Methods for Nonlinear Analysis. Theory and Applications. Singapore, World Scientific, 1996.
15. Leonov G.A., Burkin I.M., Shepeliavij A.I., Frequency Methods in Oscillation Theory. Dordrecht-Boston- London, Kluwer Academic Publishers, 1996.
16. Leonov G.A., Reitmann V., Smirnova V.B., Non-Local Methods for Pendulum-Like Feedback Systems. Stuttgart-Leipzig, Teubner Verlagsgesellschaft, 1992.
17. Буркин И.М., Леонов Г.А., Шепелявый А.И., Частотные методы теории колебаний. Изд-во Санкт-Петербургского университета, 1992.
18. Leonov G.A., Reitmann V., Attraktoreingrenzung fur nicht lineare Systeme Teubner-Texte zur Mathematik. Leipzig, 1987.
19. Гелиг А.Х., Леонов Г.А., Якубович В.А., Устойчивость нелинейных систем с неединственным состоянием равновесия. М., "Наука", 1978.

Обзоры

20. G.A. Leonov, N.V. Kuznetsov, Hidden attractors in dynamical systems. From hidden oscillations in Hilbert-Kolmogorov, Aizerman, and Kalman problems to hidden chaotic attractor in Chua circuits. Int. J. Bifurcation and Chaos, 23(1), 2013, no. 1330002

21. Леонов Г.А., Функции Ляпунова в теории размерности аттракторов. Прикл. Матем. И Механ., 76(2), 2012, 180-196.
22. G.A. Leonov, B.R. Andrievskii, N.V. Kuznetsov, A.Yu. Pogromskii, Aircraft control with anti-windup compensation, Differential equations, 48(13), 2012, 1700-1720
23. V.O. Bragin, V.I. Vagaitsev, N.V. Kuznetsov, G.A. Leonov, Algorithms for Finding Hidden Oscillations in Nonlinear Systems. The Aizerman and Kalman Conjectures and Chua's Circuits, J. Computer and Systems Sci. Int., 50(4), 2011, 511-543
24. Leonov G.A., Kuznetsov N.V., Analytical-numerical methods for investigation of hidden oscillations in nonlinear control systems**,** IFAC Proc. Volumes (IFAC-Papers Online), 18(1), 2011, 2494-2505 (survey paper 18th IFAC World Congress)
25. Леонов Г.А., Кузнецов Н.В., Кудряшова Е.В., Кузнецова О.А., Современные методы символьных вычислений: ляпуновские величины и 16-я проблема Гильберта, Труды СПИИРАН, Вып. 16, 2011, 5–36
26. G.A. Leonov, N.V. Kuznetsov, O.A. Kuznetsova, S.M. Seledzhi, V.I. Vagaitsev, Hidden oscillations in dynamical systems, Trans. Systems and Control, 2(6), 2011, 54-67
27. Леонов Г.А., Эффективные методы поиска периодических колебаний в динамических системах, Прикл. Матем. и Механ. 74(1), 2010, стр. 37-73
28. Leonov G.A., Kuznetsova O.A., Lyapunov quantities and limit cycles of two-dimensional dynamical systems. Analytical methods and symbolic computation, Regular Chaotic Dynam., 15(2-3), 2010, 354 – 377
29. Leonov G.A., Shumafov M.M., Stabilization of observable linear system, Nonlin. Dynamics and Systems Theory, 10(3), 2010, 235-268
30. Leonov G.A., Attractors, limit cycles and homoclinic orbits of low dimensional quadratic systems. Analytical methods, Canad. Appl. Math Quarterly, 17(1), 2009, 121-159
31. G. Leonov, Strange attractors and classical stability theory, Nonlin. Dynamics and System Theory, 8(1), 2008, 49-96
32. G.A. Leonov, N.V. Kuznetsov, Time-Varying Linearization and the Perron effects, Int. J. Bifurcation and Chaos, 17(4), 2007, 1079-1107
33. Г.А. Леонов, Фазовая синхронизация. Теория и приложение, Автоматика и телемеханика, N10, 2006, 47-85
34. Г.А. Леонов, Семейства трансверсальных кривых для двумерных систем дифференциальных уравнений, Вестник СПбГУ, серия 1, N4, 2006, 48-78
35. Г.А. Леонов, Странные аттракторы и классическая теория устойчивости движения, Усп. механики, 1(3), 2002, 3-43
36. Леонов Г.А., Проблема Брокетта в теории устойчивости линейных дифференциальных уравнений, Алгебра и анализ, 13 (4) , 2001, 134-155

Статьи об образовании

Леонов Г.А., О математическом образовании в России и Санкт-Петербурге. Прошлое, настоящее, будущее, Дифференц. уравнения и процессы управления, №2, 2012, стр.4-8

Г.А. Леонов, В.И. Кияев, Н.В. Кузнецов, В.В. Оносовский, С.М. Селеджи, Некоторые аспекты подготовки IT-менеджеров. Современное образование и требования ИТ менеджмента. В книге «Леонид Витальевич Канторович: математика, менеджмент, информатика»/ Под ред. Г.А. Леонова, В.С. Катькало, А.В. Бухвалова. СПб.: Изд-во Высшая школа менеджмента, 2010.